HY-330

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Introduction to telecommunication systems theory

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Coding

- Channel Coding:
  - Forward Error Correction (FEC)
  - Error-Correcting Code (ECC)

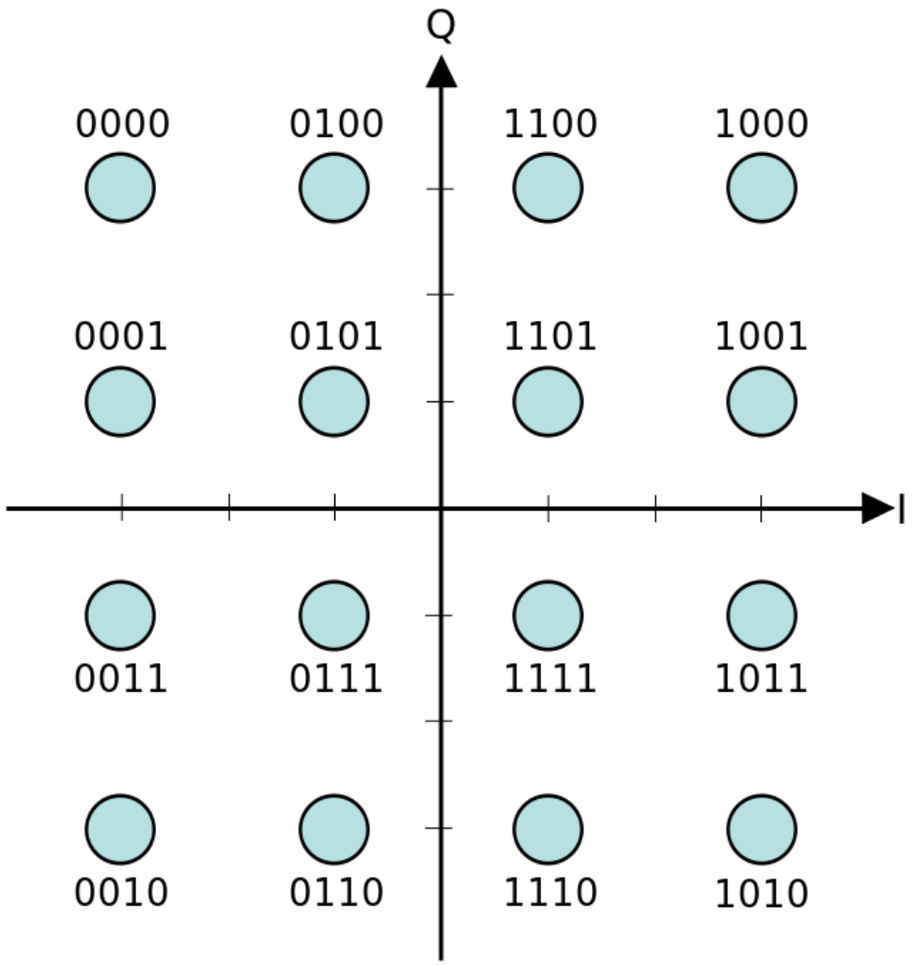
Gray Code

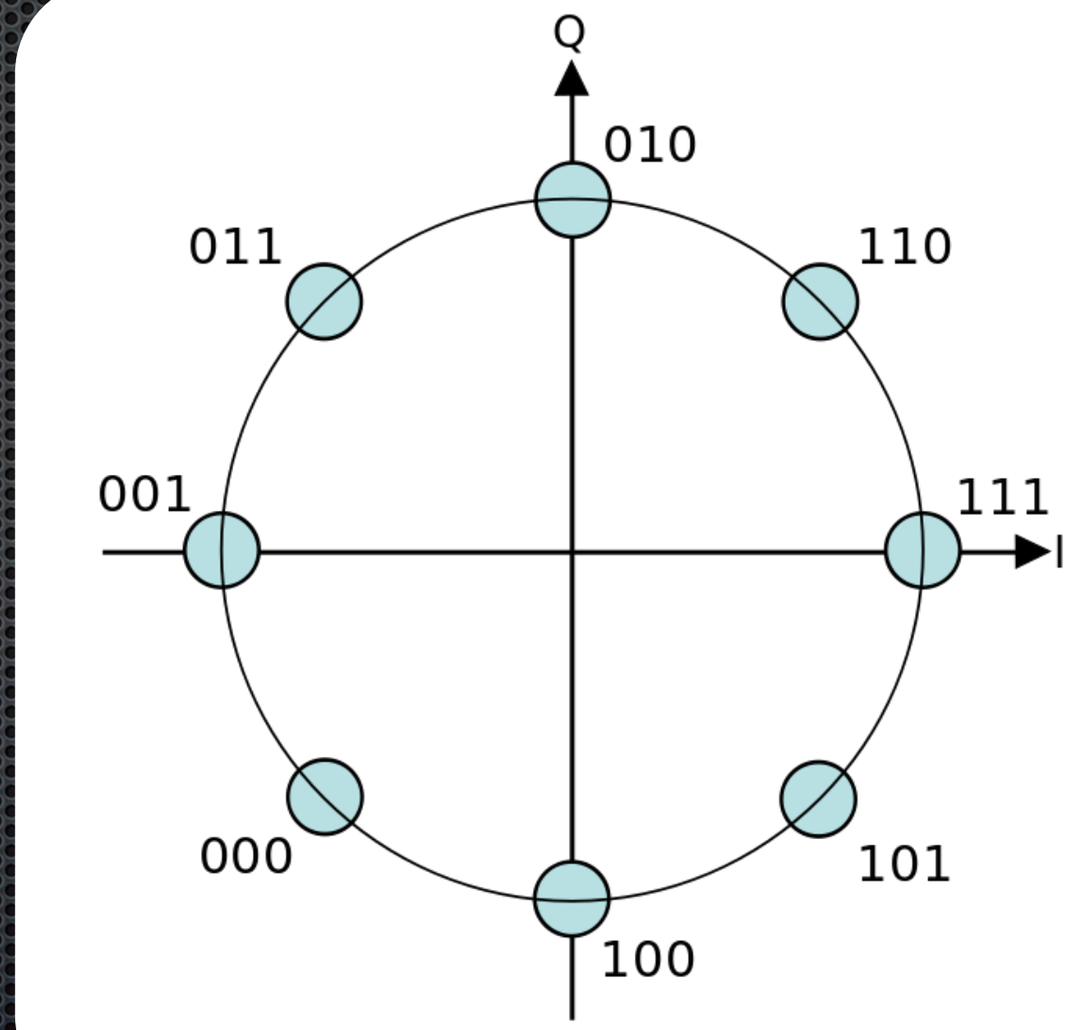
- Two successive values differ in only one bit
- Points in constellations follow grey code mapping
- Why?
  - Think of the symbol vs. bit error rate

Gray Code

Decimal	Binary	Gray
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

Constellation Diagram





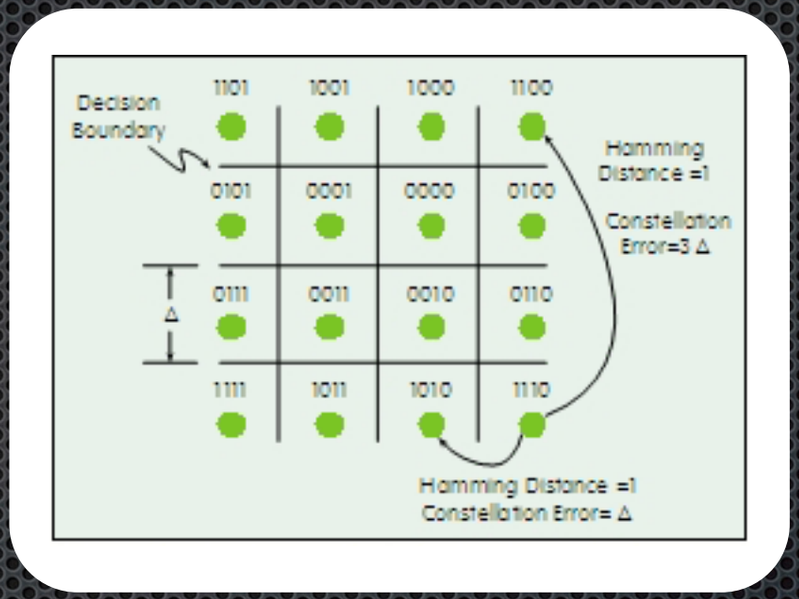
Hamming Distance

- Two strings of equal length
- The number of positions at which the corresponding symbols are different
- Examples:
  - 1101001010 - 1100010010: hamming distance = 3
  - Tooth - Booth: : hamming distance = 1

Hamming Distance

- Binary strings is a special case
- Hamming distance = XOR + count ones
- Example:
  - $1101001110 \oplus 1100010010 = 0001011100$
  - $0001011100 \Rightarrow 3 \times 1$

Hamming Distance



Error Detection

- Detect an error has occurred
- Error detection $\neq$ Error correction
- Detection can not tell the position of the error

Parity

- Make sure the number of 1 bits is either even or odd
- Add one bit to provide the parity
- Detects one (actually odd number) bit alteration

Original Data	Even Parity	Odd Parity
0 0 0 0 0 0 0 0	0	1
0 1 0 1 1 0 1 1	1	0
0 1 0 1 0 1 0 1	0	1
1 1 1 1 1 1 1 1	0	1
1 0 0 0 0 0 0 0	1	0
0 1 0 0 1 0 0 1	1	0

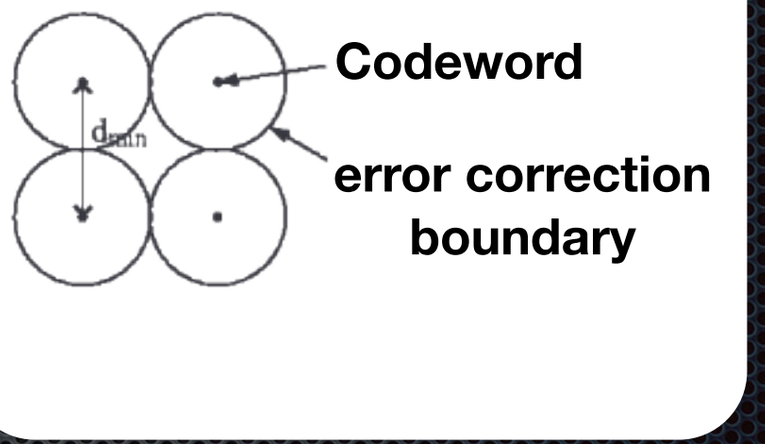
Hamming Distance

- Consider an alternate use of additional bits
- What is the hamming distance of the following codewords?

Dataword	Codeword
0 0	0 0 1
0 1	0 1 0
1 0	1 0 0
1 1	1 1 1

Hamming Distance

min hamming distance: d_{min}



- Error Detection: max $d_{min} - 1$ bit errors

- Error Correction: max $\lfloor \frac{d_{min} - 1}{2} \rfloor$ bit errors

Forward Error Correction

FEC is based on redundancy

Codes->

- systematic: original information + extra information
- non-systematic: completely different output

Forward Error Correction

Categories of Codes:

- block: use fixed-size blocks (chunks) of bits
  - hard-decision
- convolutional: are applied on continuous stream of bits
  - soft-decision
- concatenated: combined block & convolutional
  - for high erroneous channels (e.g. deep space)

Block codes

- Hamming
- Reed-Solomon
- Golay
- BCH

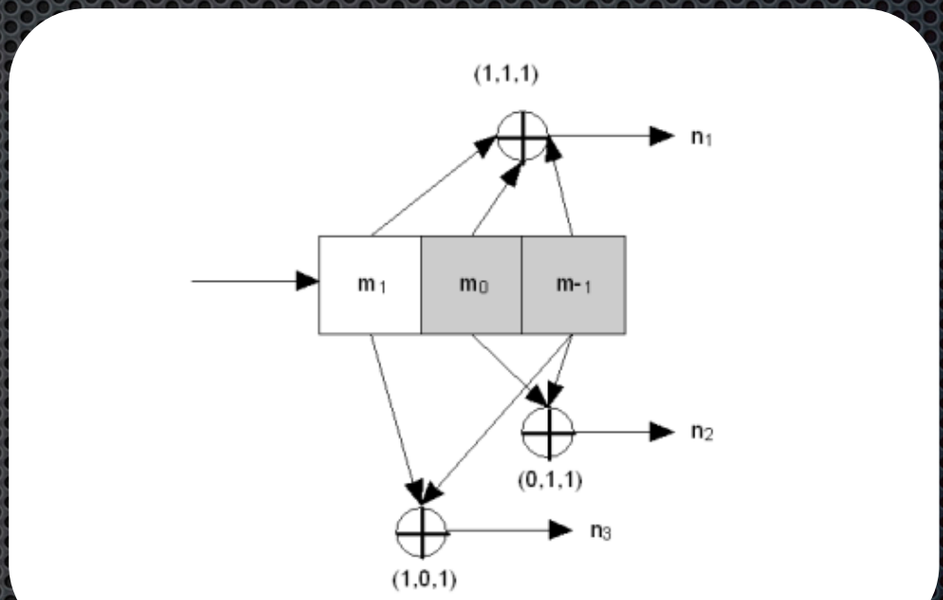
Convolutional codes

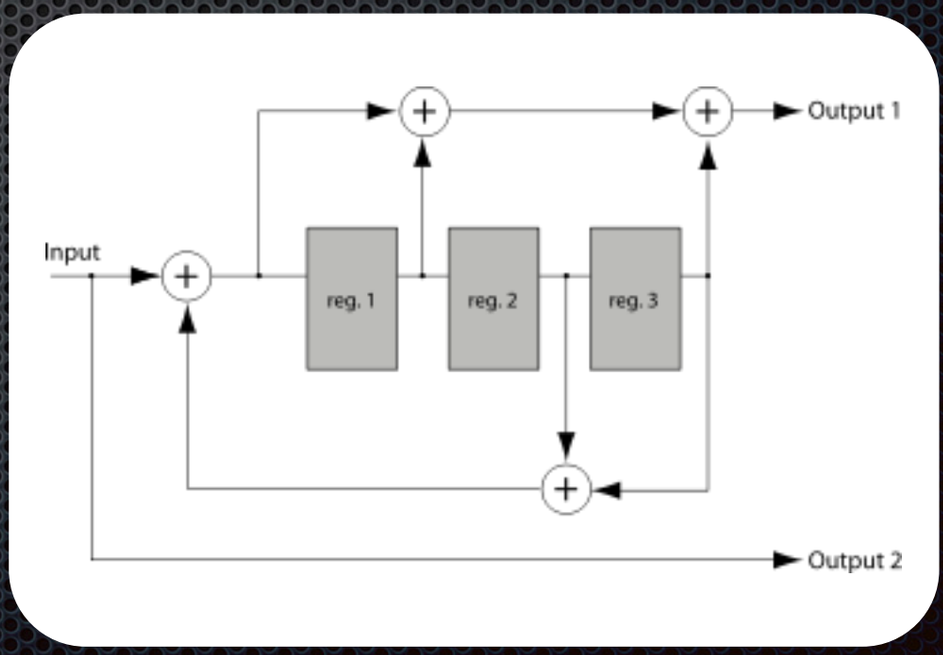
- boolean polynomial
- decoding: trellis (Viterbi)
- code rate increase: symbol puncturing

Convolutional codes

encoding:

- recursive
- non-recursive

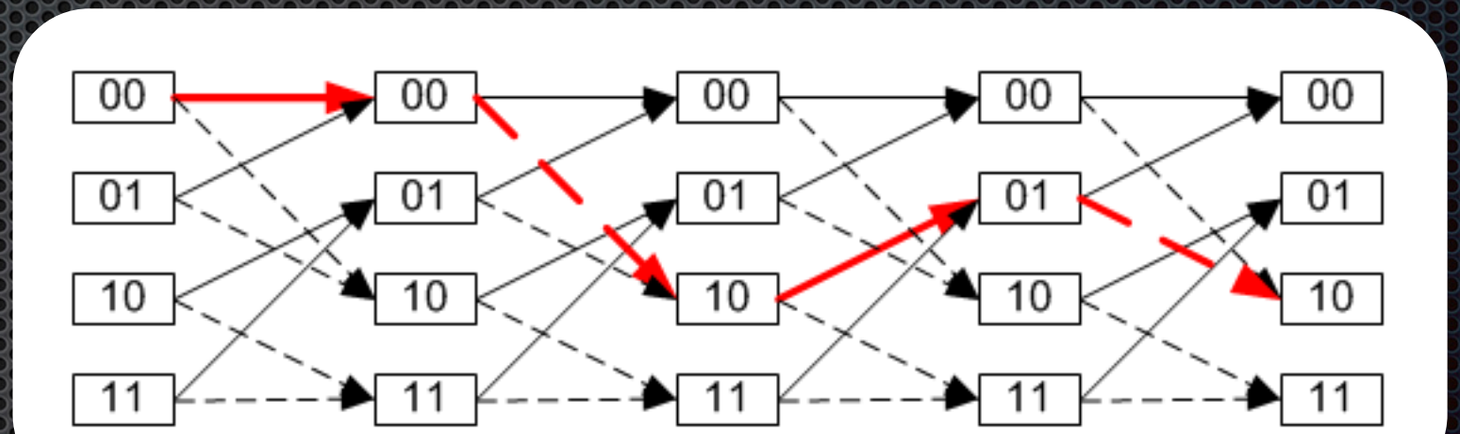




Convolutional codes

decoding:

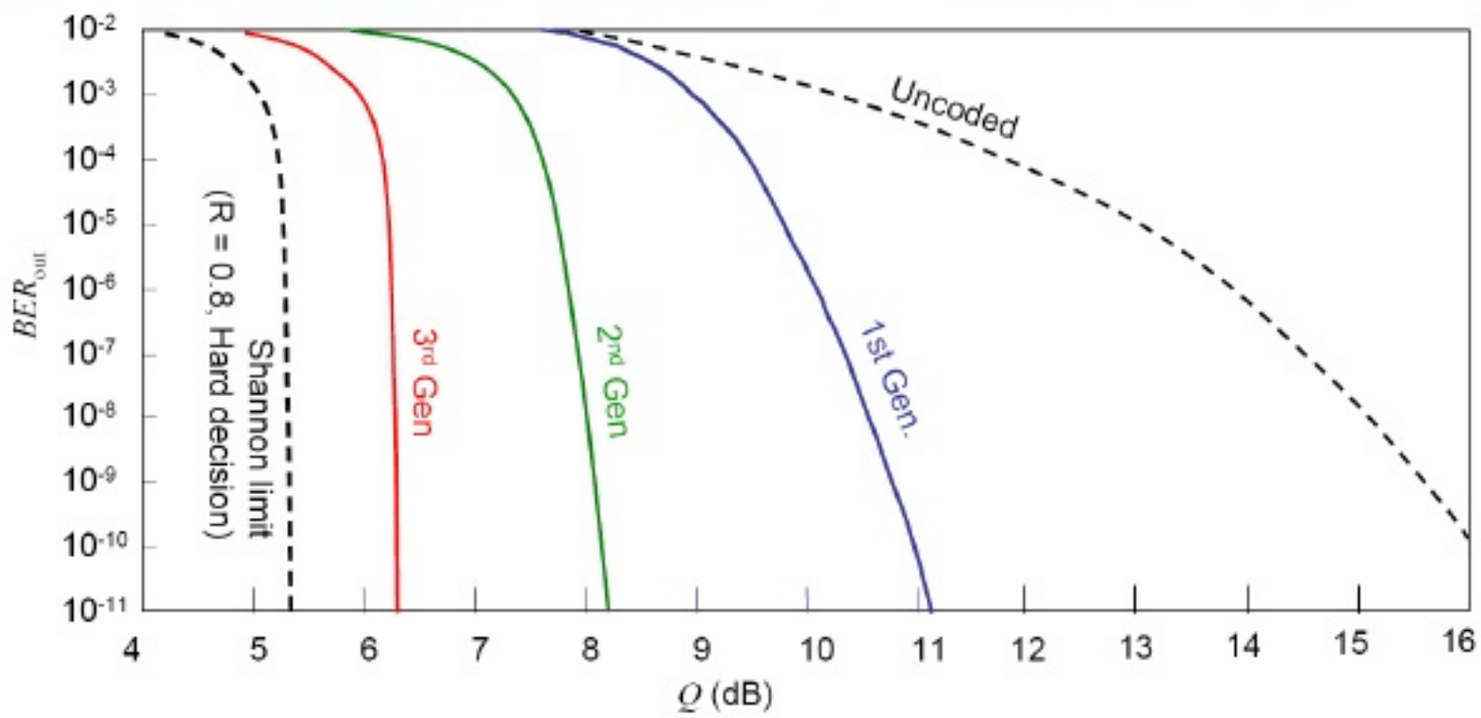
- trellis diagram
- Viterbi algorithm



Interleaving

- What is more probable to happen:
  - nicely distributed errors or bursts of errors?
- Consider this message: `ThisIsAGoodExample`
- Interleaved: `TIoxlhsoaeiAdmsGEp`
- Rx non-interleaved: `ThisIsAxxxxExample`
- Rx interleaved: `TIoxlhsxxxxAdmsGEp`
- Rx de-interleaved: `ThxsIsAGoxdExxmplx`

Coding Gain



Year	2003	2000	1993
Coding Scheme	Block Turbo Code LDPC	Concatenated RS, BCH	RS(255,239)
Net Coding Gain (@ 10^{-13})	~10dB	7~9dB	5.8dB